Fundamental Scaling Laws of Wireless Ad-hoc Networks

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1. How can we scale wireless networks?
   - Getting there with MPT + MPR (MPTR), NC(??),
   - Interference Management, New results
2. Heterogeneous Networks, New results
3. Future work
Capacity of Ad-hoc Networks

- Gupta-Kumar point-to-point communications: vanishing capacity,
- Network Coding, No gain,
- MPR + MPT (MPTR) provides optimum capacity, but limited gain considering complexity constraints,
- Distributed MIMO systems, high gains, but not practical, too much overhead,
- How can we improve the throughput further?
First we consider MIMO Broadcast in cellular networks.

- **K antennas at the base station**

- M mobile users with single antennas, M >> K

- What is the optimum capacity and how can we achieve it?

- Dirty paper Coding, C=K loglog(M), feedback=2KM real numbers

- Sharif and Hassibi: Random Beamforming, same capacity, feedback=M real numbers

- Jinal: same capacity, feedback= M integer numbers

- M goes to infinity → overhead goes to infinity
Interference Management

- Interference management achieve DPC capacity,
- Feedback = K, does not grow with M!! Perhaps the minimum anyone can claim,
- It can easily extended to ad hoc networks,
- Very practical compared to distributed MIMO systems like hierarchical MIMO systems,
- It is based on a new multiuser diversity scheme.
Interference Management

- Multiuser diversity concept was based on finding the best channel to transmit with maximum power.
- It can replace techniques such as TDMA, i.e., opportunistic beamforming.
- The new multiuser diversity concept is based on finding the best and worse channels simultaneously.
- When multiple nodes are communicating in the same channel, this technique can address the interference problem.
Interference Management

Fundamental Scaling Laws of Wireless Ad-hoc Networks
Interference Management: Approximation solution

\[ \text{SNR}_{ij} \geq \text{SNR}_{tr}, \quad \forall i, j \in 1, 2, \ldots, K, i = j \]
\[ \text{INR}_{ij} \leq \text{INR}_{tr}, \quad \forall i, j \in 1, 2, \ldots, K, i \neq j \]

\[ P(A) = \int_{\text{SNR}_{tr}}^{\infty} p(x)dx \left( \int_0^{\text{INR}_{tr}} p(x)dx \right)^{K-1} \]
\[ = e^{-\frac{\text{SNR}_{tr}}{\sigma}} \left( 1 - e^{-\frac{\text{INR}_{tr}}{\sigma}} \right)^{K-1}. \]

minimize \[ \frac{D}{K} (P(A))^{-1} \]
subject to \[ \text{SINR}_{tr} = \frac{\text{SNR}_{tr}}{(D - 1)\text{INR}_{tr} + 1} \]
The solution for $\text{INR}_{tr}^*$ is

$$\text{INR}_{tr}^* = \frac{K - 1}{D - 1} \frac{\sigma}{\text{SINR}_{tr}}.$$ 

Then the optimum value for $M$ is given by\(^2\)

$$M^* = \left[ \frac{D}{K} e^{\text{SINR}_{tr} \frac{\sigma}{\text{SINR}_{tr}}} \left( \frac{D - 1}{K - 1} \text{SINR}_{tr} e \right)^{K-1} \right].$$

Then $\text{SINR}_{tr}$ is

$$\text{SINR}_{tr}^{\text{max}} \cong \sigma \log \left( \frac{K}{D} \left( \frac{1}{e} \frac{K - 1}{D - 1} \right)^{K-1} M \right).$$
When $K = D$, then

$$M^* = \left[ e^{\frac{\text{SINR}_{tr}}{\sigma}} \left( \text{SINR}_{tr} e \right)^{K-1} \right].$$

Therefore, $K = \Theta(\log(M))$.

- Sharif and Hassibi also showed relationship between $K$ and $M$ logarithmic?
- Is this condition necessary?
Interference Management

What is the minimum feedback in MIMO broadcast channel to achieve DPC capacity?

Fig. 2. The feedback is at most $K$ with almost sure
Interference Management

Fig. 3. Simulation results for different values of SINR
Interference Management

Fig. 4. Simulation results for different fading channel environments when SINR=5dB
Interference Management

The total number of antennas $K=3$ and $K=5$

The minimum number of cooperation nodes needed $K-D$

The total number of mobile stations $M$

- $\text{SINR}_{lr}=10\text{dB}$
- $\text{SINR}_{lr}=7\text{dB}$
- $\text{SINR}_{lr}=5\text{dB}$
Interference Management

The total number of antennas $K=3$ and $K=5$

The minimum number of cooperation nodes needed vs $K D$

The total number of mobile stations $M$

Fundamental Scaling Laws of Wireless Ad-hoc Networks
Interference Management

![Graph showing the total number of mobile stations required vs. fading strength \( \sigma \). The graph includes multiple curves for different values of \( K \) and \( \text{SINR}_\Psi \).]
Interference Management

Fig. 4. The throughput capacity with and without interference management in extended wireless ad hoc network with fading channel.
Interference Management

Fig. 5. The throughput capacity with and without interference management as a function of $\sigma$, $\text{SINR}_{\text{th}}$, and $T(n)$. 
Almost all studies so far concentrated on capacity of homogeneous networks,

There are some papers exploring networks with heterogeneous node distribution,

Most military ad hoc networks include heterogeneous traffic,

What is the best strategy to achieve highest throughput capacity?
Capacity of Heterogeneous Networks

• The Separation Theorem (Using a MC-MR)

Consider a network with total available bandwidth $W$ in which $k_1$ traffic classes of equal priority are distributed uniformly in the network with each traffic class utilizing $W/k_1$ bandwidth. Separating the traffic classes and using a separate frequency and radio per node for each traffic class provides the optimum aggregate order capacity for the network.
Capacity of Heterogeneous Networks

- The area of the network is $A$.
- The total number of nodes is $n$.
- The node distribution is uniform.
- The total bandwidth of the network is $W$.
- The power of the whole network is $P$.
- The traffic distribution is a combination of $s$ unicast sessions and $k+1$ data gathering communications where $s+k=n$.
- $H_{ik}[m] = \left( \sqrt{G} \exp(j\theta_{ik}[m]) \right) / \left( d_{ik}^\alpha \right)$
- $Y_i = \sum_{k=1}^{n} H_{ik}[m]X_k[m] + Z_i[m]$
- The capacity of the network is

$$C(n) = \max_{R_{si} \text{ and } R_{dj}; i=1, \ldots, n; j=1, \ldots, s+1} \left( \min \left( \sum_{i=1}^{n} R_{si}(n), \sum_{j=1}^{s+1} R_{dj}(n) \right) \right)$$
Capacity of Heterogeneous Networks

The main result:

\[
R(n) = \\
\begin{cases}
\Omega((s + 1) \log n), & s = O \left( \left( \frac{n}{\log n} \right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}} \right), \\
\Omega \left( \frac{n}{s^{1+\varepsilon_2+\varepsilon_3} + \log n} \right), & \Omega \left( \left( \frac{n}{\log n} \right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}} \right) = s, \\
\Omega \left( \frac{s \log \log s}{\log s} + \log n \right), & \Omega \left( \left( \frac{n \log n}{\log \log n} \right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}} \right) = s, \\
\Omega \left( \frac{n}{s} + \log n \right), & \Omega \left( \left( \frac{n \log n}{\log \log n} \right)^{\frac{1}{2}} \right) = s, \\
\Omega \left( s^{1-\varepsilon_1+\varepsilon_4} + \log n \right), & \Omega \left( n^{\frac{1}{2-\varepsilon_1+\varepsilon_4}} \right) = s, \\
\Omega \left( n^{\frac{1}{2-\varepsilon_1+\varepsilon_4}} \right) = s, & = O(n) \\
\end{cases}
\]
Capacity of Heterogeneous Networks

\[ \Theta \left( \frac{s \log \log s}{\log^2 s} + 1 \right) \log n \]

\[ \Theta \left( \frac{n}{s^{1-\varepsilon_1}} + \log n \right) \]

\[ \Theta \left( \frac{n}{s} + \log n \right) \]

Many-to-One Traffic  Heterogeneous Traffic  Unicast Traffic

\[ 0 \quad S_1 \quad S_2 \quad S_3 \quad S_4 \quad \Theta(n) \]
The case of

\[ s = O \left( \left( \frac{n}{\log n} \right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}} \right) \]

- **Phase 1.** Distribution of packets from source to relays in the same cluster:

  - **r** is chosen as
    \[ \frac{\sqrt{A}}{2s} \frac{1}{2^{1+\varepsilon_2+\varepsilon_3}} \]
  - or
    \[ \frac{\sqrt{A}}{2n} \frac{1}{(1+\varepsilon_2)\beta_1} \]

  The aggregate throughput of this phase is
  \[ s(K_4 \log n + K_5) \]

  The time needed is
  \[ \frac{K_6K_2L}{s^{1+\varepsilon_2+\varepsilon_3}} \]

  or
  \[ \frac{K_6K_2L}{(1+\varepsilon_2)\beta_1} \frac{n}{K_4 \log n + K_5} \]
The case of Phase 2. MIMO Cooperation Transmission:

The aggregate throughput of this phase is

\[ s = O \left( \left( \frac{n}{\log n} \right)^{\frac{1}{2+\varepsilon_2+\varepsilon_3}} \right) \]

Or

\[ K_7 \frac{n}{s^{1+\varepsilon_2+\varepsilon_3}} \]

\[ K_7 \frac{n}{n^{(1+\varepsilon_2)\beta_1}} \]

The time needed in this phase is

\[ \frac{C_1}{K_7} s \]
The case of Phase 3. Transmission from Relays to Destination:

The aggregate throughput of this phase is the same as that in Phase 1.

The time needed in this phase is

\[
\frac{K_6 C_2 C_3 n}{K_5 + K_4 \log n} \quad \text{or} \quad \frac{K_6 C_2 C_3 n}{K_5 + K_4 \log n}
\]
The case of $s = O \left( \left( \frac{n}{\log n} \right)^{2+\varepsilon_2+\varepsilon_3} \right)$

- The lower bound of this case is

$$R_1(n) = \frac{K_2 L s^{n}}{s^{1+\varepsilon_2+\varepsilon_3}} - \frac{K_6 K_2 L s^{n}}{K_4 \log n + K_5} + \frac{C_1 s}{K_7} + \frac{K_6 C_2 C_3 s^{n}}{K_5 + K_4 \log n}$$

or

$$\geq K_8 s \log n.$$

$$R_1(n) = \frac{K_2 L s^{n}}{n^{(1+\varepsilon_2)\beta_1}} - \frac{K_6 K_2 L s^{n}}{K_4 \log n + K_5} + \frac{C_1 s}{K_7} + \frac{K_6 C_2 C_3 s^{n}}{K_5 + K_4 \log n}$$

or

$$\geq K_8 s \log n.$$
Capacity of Heterogeneous Networks

\[
R(n) = \Theta\left(\frac{s \log \log s}{\log^2 s} + 1\right) \log n
\]

\[
R_4
\]

\[
R_3
\]

\[
R_2\quad R_1
\]

\[
\Theta((s+1) \log n)
\]

\[
\Theta\left(\frac{n}{s^{1-\varepsilon_3} + \log n}\right)
\]

\[
\Theta\left(\frac{n}{s^{1-\varepsilon_4} + \log n}\right)
\]

\[
\Theta\left(n^{1-\varepsilon_1}\right)
\]

\[
\Theta(n)\quad S
\]

Many-to-One Traffic  Heterogeneous Traffic  Unicast Traffic

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Future Work

- Capacity of heterogeneous networks?
- How can we make the interference management more practical, i.e., reducing $m$?
- How can we relate wireless heterogeneous networks to social networks in our analysis?