Analog Network Coding in High-SNR Relay Networks

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Outline

- Introduction
- Unicast
- Multi-source networks
- Extensions
Network Coding Success

- Achieves multicast capacity in noiseless networks
  \[\text{[Ahlswede, Cai, Li & Yeung, 2000]}\]
- Linear network coding suffices
  \[\text{[Li, Yeung & Cai, 2003], [Koetter & Médard, 2003]}\]
- Relay sends a linear combination of received packets
Relay receives a sum of sent signals + noise

\[ y_R = h_1 x_1 + h_2 x_2 + n \]

Decoding at the relay reduces rates
Wireless Networks

- Relay receives a sum of sent signals + noise
  \[ y_R = h_1 x_1 + h_2 x_2 + n \]
- Decoding at the relay reduces rates
- Network coding on physical layer
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- Decoding at the relay reduces rates
- Network coding on physical layer
- Compute-and-forward [Nazer, Gastpar, 2007]
Relay receives a sum of sent signals + noise

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Decoding at the relay reduces rates

Network coding on physical layer

Compute-and-forward [Nazer, Gastpar, 2007]

Amplify-and-forward / Analog network coding [Katti, Marić, Goldsmith, Katabi, Médard, 2007]

\[ x_R = \beta(h_1 x_1 + h_2 x_2 + n) \]
Relay receives a sum of sent signals + noise

\[ y_R = h_1 x_1 + h_2 x_2 + n \]

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Network coding on physical layer

Compute-and-forward \cite{Nazer, Gastpar, 2007}

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\[ x_R = \beta(h_1 x_1 + h_2 x_2 + n) \]

High-SNR performance - What is it?
Unbounded Capacity Gap in High-SNR

- Diamond network [Schein, 2001]
Unbounded Capacity Gap in High-SNR

- Diamond network \([\text{Schein}, 2001]\)
- As \(a\) increases: the gap between analog network coding and cut set increases \([\text{Avestimehr, Diggavi & Tse, 2009}]\)
Cut-set Upper Bound vs. Achievable Rate

- For: a large, $P_s = P_1 = P_2 = 1$

- Cut-set upper bound:
  $$C \leq 3 \log a$$

- Achievable rate:
  $$R = \left( h_{1D} \sqrt{P_1} + h_{2D} \sqrt{P_2} \right)^2$$
  $$\frac{h_{1D}^2 P_1}{h_{s1}^2 P_s} + \frac{h_{2D}^2 P_2}{h_{s2}^2 P_s} + 1$$
  $$R = 2 \log a$$

- A gap due to noise amplification for high channel gains
- Unlike in the point-to-point channel, high channel gains do not guarantee the high-SNR regime
Appropriate way to reach high SNR when gains are given (e.g., fixed topology)
Scaling Behavior

As transmit powers increase the analog coding rate scales as the cut-set bound

Constant gap
Multihop Network

- Noise propagated over multiple hops
- Very little analysis on multihop amplify-and-forward
  - Degrees of freedom and diversity-multiplexing tradeoff analyzed [Borade, Zheng and Gallager, 2007]
- What are high-SNR conditions?
Multihop Network

- Noise propagated over multiple hops
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- What are high-SNR conditions?
- Assumptions: layered network, directed links, no fading
- A layered relay network:

\[ y_k = \sum_{j \in \mathcal{N}(k)} h_{jk} x_j + z_k \]

\( \mathcal{N}(k) \) - set of neighbors transmitting to node k
Cut-set Bound

Denote received power at destination $D$:

$$P_D = \left( \sum_{i \in \mathcal{N}(D)} h_{iD} \sqrt{P_i} \right)^2$$

$\mathcal{N}(D)$ - set of neighbors transmitting to node $D$

MAC cut-set upper bound at the destination node:

$$C_{\text{MAC}} = \frac{1}{2} \log (1 + P_D)$$
Analog Network Coding: High-SNR Performance

Received power at any relay $k$:

$$P_{R,k} = \left( \sum_{j \in \mathcal{N}(k)} h_{jk} \sqrt{P_j} \right)^2$$

We define high-SNR conditions as:

$$P_{R,k} \geq 1/\delta \quad \text{for each } k \neq D.$$
Analog Network Coding: High-SNR Performance

Received power at any relay $k$:

$$P_{R,k} = \left( \sum_{j \in N'(k)} h_{jk} \sqrt{P_j} \right)^2$$

We define high-SNR conditions as:

$$P_{R,k} \geq \frac{1}{\delta} \quad \text{for each } k \neq D.$$  

We show that analog network coding achieves

$$R = \frac{1}{2} \log \left( 1 + \frac{1}{(1 + \delta)^{L-1}} \frac{P_D}{P_{Z,D} + 1} \right)$$

$P_D$ - received power at the destination
$P_{Z,D}$ - accumulated noise at the destination
Analog Network Coding: High-SNR Performance

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- Examine the red term behavior in high-SNR
In High-SNR

- High received SNR at each relay $k$:
  \[ \text{SNR}_k \geq \frac{1}{\delta} \]

- We show that:
  - Propagated noise at each layer is of order $\delta$
  - Propagated noise at destination from $L$ layers:
  \[ P_{Z,D} \leq L \delta P_D \]
High-SNR Performance

- Achievable rate:

\[ R = \frac{1}{2} \log \left( 1 + \frac{1}{(1 + \delta)^{L-1}} \frac{P_D}{P_{Z,D} + 1} \right) \]

- Accumulated noise satisfies:

\[ P_{Z,D} \leq L\delta P_D \]

- It follows that:

1. When \( \delta \to 0 \):

   \[ P_{Z,D} \to 0 \]

   Analog network coding achieves capacity

2. When \( P_D \) increases as \( \delta \to 0 \) such that \( P_D\delta = \text{const.} \):

   Analog network coding is within constant gap from the upper bound
Analog network coding and the MAC cut-set bound

- $P_1 = P_2 = 100$
- $P_3 = P_4 = 10$
- $h_{ij} = 10$, for all $i,j$

3-hop Example

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Analog Network Coding in High-SNR Relay Networks
3-hop Example: Scaling Behavior

- All transmit powers increase at same rate
- A constant gap between the analog coding rate and the cut-set bound
Multicast? Multiple Sources?

Relays forward sum of signals carrying multiple data streams
Multicast Example

- Two sources $S_1$ and $S_2$ wish to send their messages to two destinations $D_1$ and $D_2$. 

![Diagram showing network with sources $S_1$ and $S_2$, destinations $D_1$ and $D_2$, and intermediate nodes 3 and 4 with channel gains $h_{13}, h_{35}, h_{23}, h_{24}, h_{45}, h_{46}$.]

Analog network coding and the MAC cut-set bound for multicast

- $P_1 + P_2$ [bits/channel use]

- $P_3 = P_4 = 10$
- $h_{13} = 1$
- $h_{14} = h_{23} = h_{24} = 10$
- $h_{35} = h_{45} = 5$

- Amplified noise power

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Analog Network Coding in High-SNR Relay Networks
Multicast in High SNR

- Extends to larger layered networks
Multicast in High SNR

- Extends to larger layered networks
- Propagated noise at each layer is of order $\delta$
- The analog network coding loss is of order $\delta$
- Additionally, lose at most $1/2$ bit
- Conditions slightly changed compared to the relay networks
Conclusions

▶ Results suggest a suitable, simple strategy for high-SNR
▶ Characterize regime in which noise is negligible
  ▶ When received powers are bounded by $1/\delta$, the noise is bounded by $\delta$
▶ Analog network coding rate approaches cut-set bound as received relay powers increase
▶ The rate is within a constant gap from the cut-set bound as all the transmit power increase
Extensions

- Analog network coding approaches capacity in high SNR
  - Diamond network
  - Multihop network
- Layered networks, directed links, full-duplex relays, no fading
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- Wireless links
  - Loops, half-duplex relays, fading
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